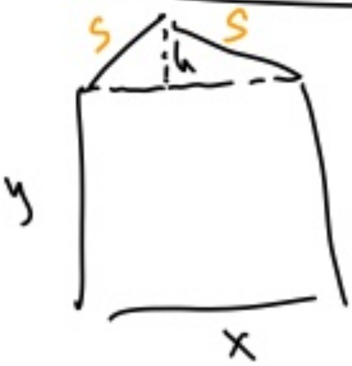
Calculus Review

① $\int_0^2 x^2\,dx =$

② $\int_0^\pi \sin^2(t)\,dt =$

Test #2 corrections
- can turn in multiple times by <u>Apr. 16.</u>



Perimeter fixed

$$P(x,y,h) = x + 2y + 2\sqrt{h^2 + x^2/4}$$

constraint

$$\text{Area} = A(x,y,h) = xy + \tfrac{1}{2}xh$$

$$\nabla A = \lambda \nabla P$$

$$P = \text{const.}$$

$$x + 2y + 2\sqrt{h^2 + x^2/4} = P = \#$$

$$y + \frac{h}{2} = \lambda\left(1 + \frac{x}{2\sqrt{h^2+x^2/4}}\right)$$

$$x = \lambda 2$$

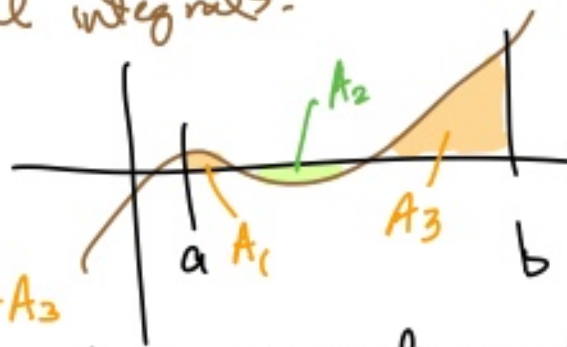
① $\int_0^2 x^2\,dx \overset{FTC}{=} \left.\frac{x^3}{3}\right|_0^2 = \boxed{\frac{8}{3}}$

② $\int_0^\pi \sin^2(t)\,dt = \int_0^\pi \left(\frac{1}{2} + \frac{1}{2}\cos(2t)\right)dt$

$$= \left.\frac{t}{2} + \frac{1}{4}\sin(2t)\right|_0^\pi = \boxed{\frac{\pi}{2}}$$

Review of 1-dimensional integrals.

$$\int_a^b f(x)\,dx =$$



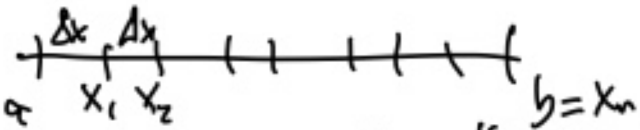
$$= A_1 - A_2 + A_3$$

signed area under $y = f(x)$ from $x=a$ to $x=b$.

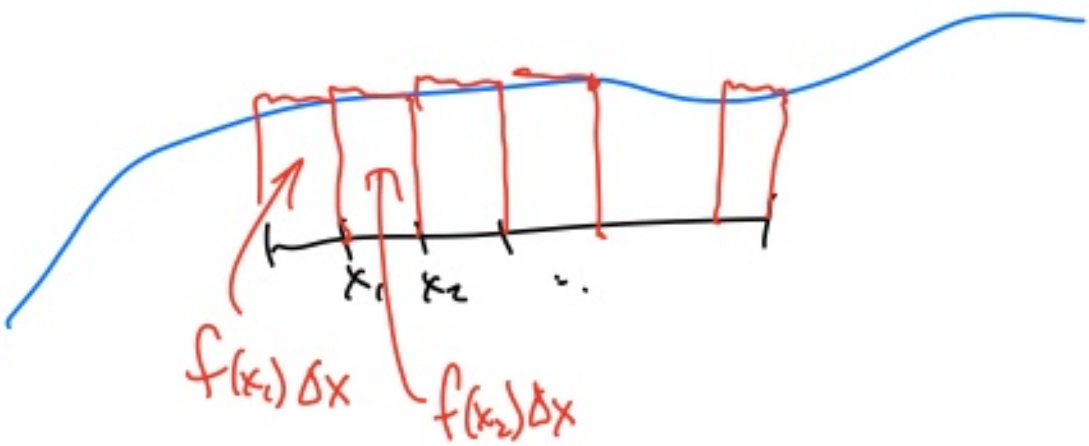
Defn. $$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\,\Delta x$$

$$\Delta x = \frac{b-a}{n}$$

$$x_k = a + k\Delta x$$

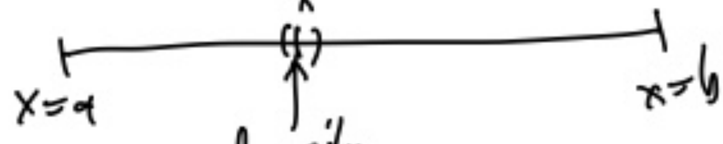


n subintervals



(density)$\cdot$(volume) = mass

rod



density $= f(x)$

$$\text{mass of rod} = \int_a^b f(x)\,dx$$

Tool to Calculate Integrals:

$$\int_a^b F'(x)\,dx = F(b) - F(a) \quad (\text{FTC})$$

Fundamental Theorem of Calculus.

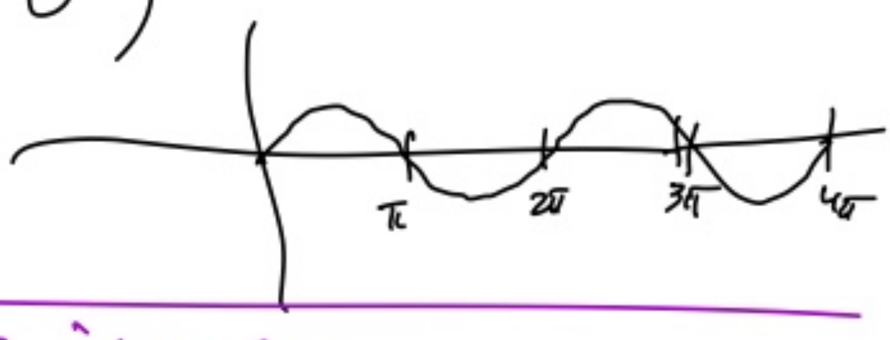
Average value of a function $f(x)$ on $[a,b]$

$$= \lim_{n\to\infty} \frac{\sum_{k=1}^{n} f(x_k)}{n} \; \cdots$$

$$= \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\Rightarrow \int_a^b f(x)\,dx = (b-a)\cdot\left(\text{average value of } f(x) \text{ on } [a,b]\right)$$

$$\left(\int_0^{4\pi} \sin(x)\,dx = 0\right)$$



Higher Dimensional integrals.

① $$\int_{y=0}^{1}\int_{x=0}^{\pi} \sin(xy)\,dx\,dy$$

$$= \int_{y=0}^{1} \underbrace{\left(\int_{x=0}^{\pi} \sin(xy)\,dx \right)}_{\text{\# that depends on } y} dy$$

y is constant

\# that depends on y

i.e. a function of y

$$= \int_{y=0}^{1} \left(-\frac{\cos(xy)}{y} \right) \Big|_{x=0}^{\pi} dy$$

$$= \int_{y=0}^{1} \left(-\frac{\cos(\pi y)}{y} - \frac{-\cos(0)}{y} \right) dy$$

$$= \int_{0}^{1} \frac{1 - \cos(\pi y)}{y}\, dy \overset{\text{numerically}}{=} \boxed{1.648}.$$

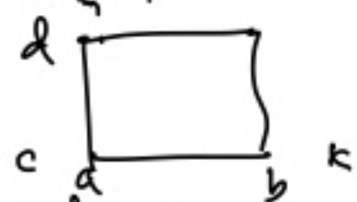
9:31 AM Fri Mar 8

sagecell.sagemath.org

Type some Sage code below and press Ev

```
f(y)=(1-cos(pi*y))/y
ans = integral(f(y),y,0,1)
show(ans.n())
```

Thm (Fubini Thm) – If $|f(x,y)|$ is integrable over the rectangle $[a,b]\times[c,d]$



Then $$\int_a^b \int_c^d f(x,y)\,dy\,dx = \int_c^d \int_a^b f(x,y)\,dx\,dy.$$

$\boxed{\text{Ex}}$ $$\int_0^1 \underbrace{\int_0^x xe^{tx}\,dt}_{\substack{\text{bounds depend on } x\\ \text{integral depends on } x\\ \Rightarrow \text{answer depends on } x.}}\,dx$$

$$= \int_{x=0}^1 \left(\int_{t=0}^x xe^{tx}\,dt \right) dx$$

$$= \int_{x=0}^1 \left(e^{tx} \Big|_0^x \right) dx$$

$$= \int_{x=0}^1 \left(e^{x^2} - e^0 \right) dx$$

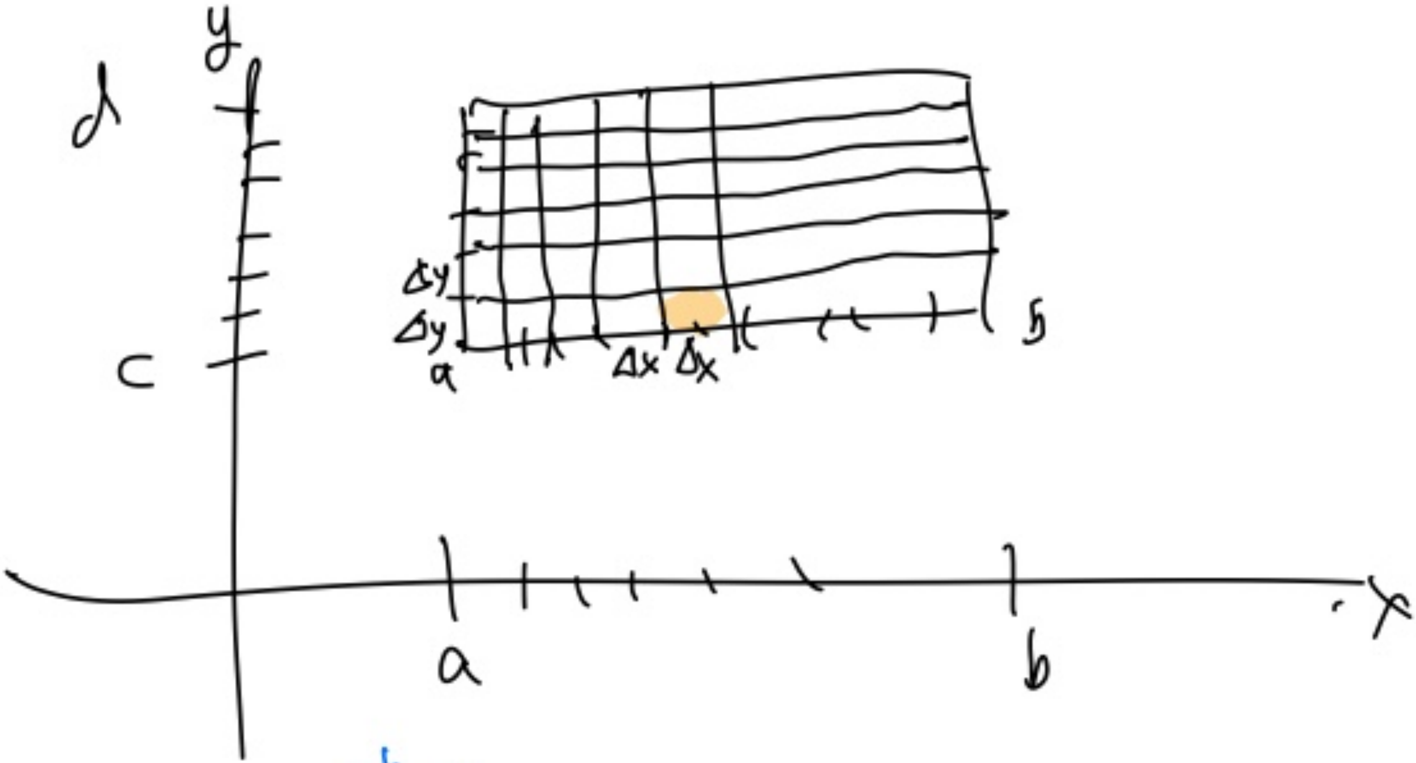
$$= \int_0^1 \left(e^{x^2} - 1 \right) dx$$

$$\boxed{= 0.463}$$ <u>numerics</u>

What is the integral?

$$\int_a^b \left(\int_c^d f(x,y)\,dy \right) dx$$

$$= \lim \sum_{k,l} f(x_k, y_l)\,\Delta y\,\Delta x$$

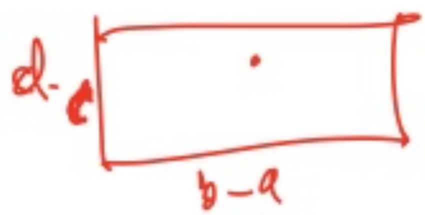


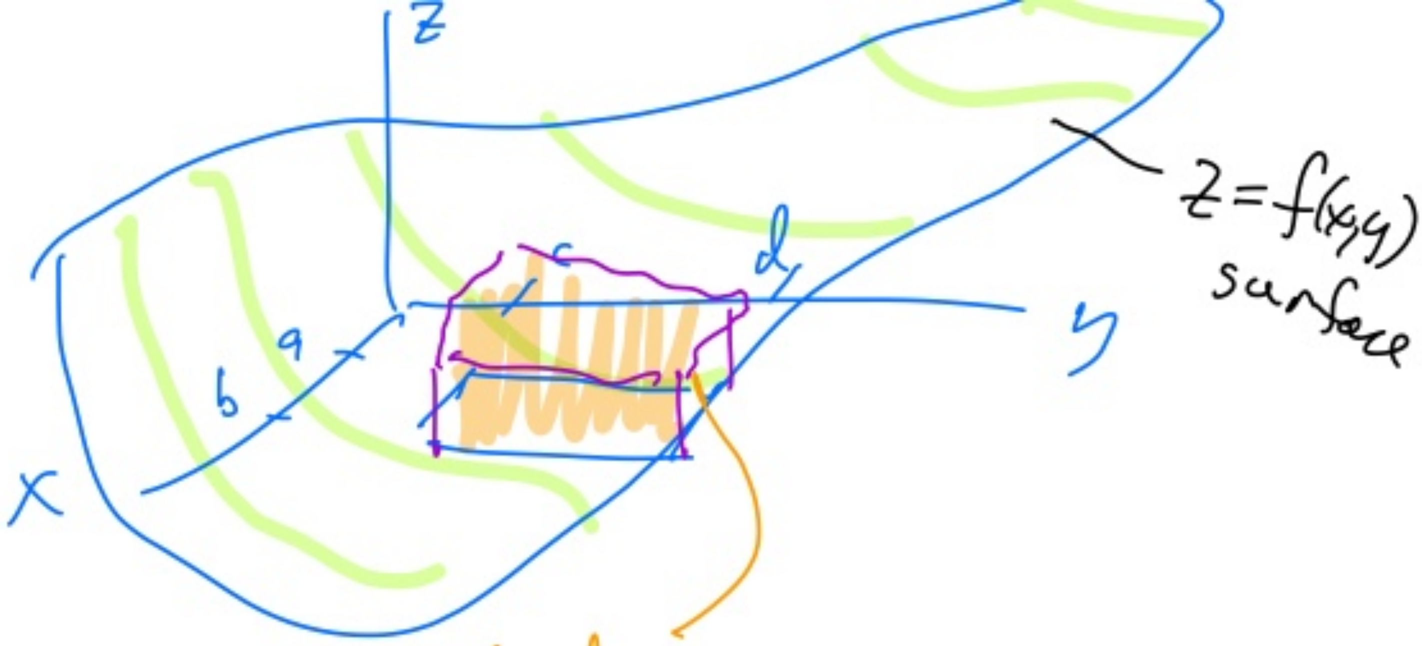
$$\int_a^b \int_c^d f(x,y)\, dy\, dx$$

$$= \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \text{sum of } \Big(\underbrace{f(x,y)}_{\substack{\text{function} \\ \text{value} \\ \text{at a} \\ \text{point}}} \cdot \underbrace{\Delta x \, \Delta y}_{\substack{\text{area of} \\ \text{small} \\ \text{box}}} \Big)$$

If $f(x,y)$ = density of the plate at position (x,y).

$$\int_a^b \int_c^d f(x,y)\, dy\, dx = \text{mass of the plate}$$





$$\int_a^b \int_c^d f(x,y)\,dy\,dx$$

= signed volume under $z=f(x,y)$
over the rectangle
$[a,b]\times[c,d]$

in xy plane.